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Fault diagnosis for T-S systems in finite frequency domain : Some Results and Perspectives

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Outline



Contexte and Motivation

- Takagi-Sugeno modeling
- Unknown Input Observers : LMI design,
- Limitations: Full frequency approach

Finite Frequency Domain Approach

Objectives & Motivation

Multiobjective synthesis : UIO-FFD

- Sensitivity
- Robustness

Perspectives

Nonlinear Systems : LPV, T-S Fuzzy Systems

$$\begin{cases} \sigma x(t) = f(x(t), u(t)) \\ y(t) = h(x(t)) \end{cases} \Rightarrow \begin{cases} \sigma x(t) = \sum_{i=1}^{n} \mu_i(\xi) \left(A_i x(t) + B_i u(t) \right) \\ y(t) = \sum_{i=1}^{n} \mu_i(\xi) C_i x(t) \end{cases} \quad \sigma x(t) = \dot{x}(t) \\ \sigma x(t) = \dot{x}(t) \\ \sigma x(t) = \dot{x}(t) \end{cases}$$



Nonlinear Systems : LPV, T-S Fuzzy Systems

$$\begin{cases} \sigma x(t) = f(x(t), u(t)) \\ y(t) = h(x(t)) \end{cases} \Rightarrow \begin{cases} \sigma x(t) = \sum_{i=1}^{n} \mu_i(\xi) \left(A_i(x) x(t) + B_i(x) u(t) \right) \\ y(t) = \sum_{i=1}^{n} \mu_i(\xi) C_i(x) x(t) \end{cases}$$

Nonlinear Systems ---- > Polynomial T-S Systems

Polynomial T-S Systems : SOS Tool

- L. Zadeh. Outline of a new approach to the analysis of complex system and decision process. IEEE transaction on Systems Man and Cybernetic-part C, 3(1), p. 28-44, 1973.
- T. Takagi, M. Sugeno. Fuzzy identification of systems and its application to modelling and control. IEEE Trans. on Systems, Man, Cybernetics, vol. 15, no.1, p. 116-132,1985.

Nonlinear Systems : LPV, T-S Fuzzy

$$\begin{cases} \sigma x(t) = f(x(t), u(t)) \\ y(t) = h(x(t)) \end{cases} \Rightarrow \begin{cases} \sigma x(t) = \sum_{i=1}^{n} \xi_{i}(\theta) \left(A_{i}x(t) + B_{i}u(t)\right) \\ y(t) = \sum_{i=1}^{n} \xi_{i}(\theta)C_{i}x(t) \end{cases}$$
$$\underbrace{\mathsf{LPV}}_{\substack{i=1\\\xi_{i}(\theta) \ge 0}} \mathsf{T-S} \qquad \underbrace{\mathsf{Switching}}_{\substack{\xi_{i}(\theta) \in \{0,1\}}} \end{cases}$$

- 1. Controller & Observer Design
- 2. FDI-FTC
 - > Numerical tools : LMI & SOS
 - Reduction of conservatism

M. Chadli & P. Borne. Multiple Models Approach in Automation: Takagi-Sugeno Fuzzy Systems. Wiley. p. 208. 2013.



Nonlinear Systems : Contexte





How Maintaining Security in the Presence of Faults: Example

- Integration of contraintes s.t. saturation, delays,
- Reconfiguration of control laws

D Saifia, M Chadli, et al. Robust H∞ static output feedback control for discrete fuzzy systems with actuator saturation via fuzzy Lyapunov functions. Asian Journal of Control, 2019.

LMI design conditions

TS Systems: $\begin{cases} \dot{x}(t) = \sum_{i=1}^{n} \mu_i(\xi) \left(A_i x(t) + B_i u(t) + R_i \overline{u}(t) \right) \\ y(t) = C x(t) + F \overline{u}(t) \end{cases}$

Unknown Input Observer (UIO) :

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{n} \mu_i(\xi) \left(\frac{N_i z(t)}{t} + \frac{G_i u(t)}{t} + \frac{L_i y(t)}{t} \right) \\ \hat{x}(t) = z(t) - \frac{E}{t} y(t) \end{cases}$$

$$\begin{array}{lll} \textbf{Objective:} & N_i, \ G_i, \ L_i, \ E \ ? \\ & s.t. & \lim_{t \to \infty} \left(\hat{x}(t) - x(t) \right) \to \ 0 \end{array}$$

LMI design conditions

$$\begin{cases} XA_i + SCA_i - W_iC + (XA_i + SCA_i - W_iC) < 0\\ SF = 0\\ (X + SC)R_i = W_iF \end{cases}$$

Observer parameters:

$$\begin{cases} E = X^{-1}S, \\ G_{i1} = (I + EC)B_i \\ N_i = (I + EC)A_i - X^{-1}W_iC \\ L_i = X^{-1}W_i - N_iE \end{cases}$$

Performances : LMI regions

Unknown Input Observer

LMI design conditions

Extensions : Unmeasurables variables, Disturbances

1. Unmesurable variables : continuous/discrete-time cases

$$\mu_{i}(\theta) \Rightarrow \mu_{i}(\hat{\theta}): \begin{cases} \sigma x(t) = \sum_{i=1}^{n} \mu_{i}(\hat{\xi}) \left(A_{i}x(t) + B_{i}u(t) + R_{i}\overline{u}(t) + H_{i}w(t) \right) \\ y(t) = \sum_{i=1}^{n} \mu_{i}(\hat{\xi}) \left(C_{i}x(t) + F_{i}\overline{u}(t) + J_{i}w(t) \right) \end{cases}$$

2. Polynomial TS system (SOS) :

$$\begin{cases} \sigma x(t) = \sum_{i=1}^{n} \mu_i(\xi) \left(\frac{A_i(x)x(t)}{A_i(x)} + \frac{B_i(x)u(t)}{B_i(x)} + \frac{R_i(x)\overline{u}(t)}{W(t)} \right) \\ y(t) = Cx(t) + F\overline{u}(t) + Jw(t) \end{cases}$$

Unknown Input Observer

Other approaches:

- Estimation of state and unknown inputs (faults)
- Uncertainties $\left. \begin{array}{c} y(t) = (C + \Delta C)x(t) \end{array} \right.$
- Lyapunov Functions : $P \rightarrow P_i, \dots$

•
$$\lim_{t \to \infty} \sup \left\| \hat{x}(t) - x(t) \right\| \leq \varepsilon \left(au \ lieu \ \lim_{t \to \infty} \left\| x(t) - \hat{x}(t) \right\| = 0 \right)$$

- estimation of f(t): polynomial form or any faults
 - $\rightarrow descripteur \ approach : x(t) \Rightarrow \overline{x}(t) = \begin{bmatrix} x(t)^T, & f(t)^T \end{bmatrix}^T$
 - \rightarrow PI observateur
- finite time estimate: $\hat{x}(t) x(t) = 0$ for $t \ge \pi$

M. Chadli, A. Abdo, S. Ding. $H-/H^{\infty}$ Fault Detection Filter Design for Discrete-time Fuzzy System. **Automatica** 2013.

Unknown Input Observer

Vehicle application: Experimental validation

Cryptography : Chaos synchronisation

M. Chadli, I. Zelinka, T. Youssef. Unknown inputs observer design for fuzzy systems with application to chaotic system reconstruction. Computers & Mathematics with Applications 66 (2), 147-154.

Unknown Input Observer : Limitation

- Findings :
 - ✓ Observer for full frequency domaine only
 - ✓ Results do not take into account the frequency domain of the signals: Conservatism

Finite Frequency Domain Approach

Objectives

- □ Incorporate the frequency range of the external signals into the synthesis conditions.
- **Robust Estimation Finite Frequency Domain (FFD)**

PhD thesis of A. Chibani (Nov 2016) : IEEE-TFS 2018, Automatica 2018

Finite Frequency Domain Approach

• The finite frequency method generally leads to more efficient results and less conservative conditions.

according to the frequency of the external signal

T. Iwasaki, S. Hara. Generalized KYP lemma: unified frequency domain inequalities. IEEE TAC 2005.

Finite Frequency Domain Approach

Motivation : Vertical vibrations in frequencies between 4 Hz and 8 Hz are the most sensitive range for the human body. (Norm ISO2361)

Objectives :

- □ Incorporate the frequency range of the external signals into the synthesis conditions.
- **Robust Estimation Finite Frequency Domain (FFD)**

• Illustrative Example

- ✓ Frequency of signal d(t): $\omega_d \in \Omega_d^m = \{\omega_d \in \mathbb{R} \mid \omega_{d_1} \le \omega_d \le \omega_{d_2}\}$
- ✓ Frequency of signal f(t): $\omega_f \in \Omega_f^l = \{\omega_f \in \mathbb{R} \mid |\omega_f| \le \omega_{f_l}\}$

Multiobjective synthesis : Preliminary results

• For $d \neq 0$, $H \infty$ performance: $||r(t)||_2 < \gamma ||d(t)|_2$

$$\begin{cases} \dot{e}(t) = \sum_{i=1}^{r} \mu_i \left(\xi(t)\right) \left(\mathcal{A}_i e(t) + \mathcal{B}_i d(t)\right) \\ r(t) = \mathcal{C}e(t) + \mathcal{D}d(t) \end{cases}$$

• Frequency of signal d(t): $\omega_d \in \Omega_d^m = \{\omega_d \in \mathbb{R} \mid \omega_{d_1} \le \omega_d \le \omega_{d_2}\}$

<u>Lemma 1</u>.

The H ∞ performance: $||r(t)||_2 < \gamma ||d(t)|_2$ is garanteed in the middle frequency band $(\omega_{d_1} \le \omega_d \le \omega_{d_2})$ if there exist symmetric matrices P and Q > 0 such that:

Multiobjective synthesis : Preliminary results

• For f≠0, H_performance: $||r(t)||_2 > \beta ||f(t)||_2$

$$\begin{cases} \dot{e}(t) = \sum_{i=1}^{r} \mu_i \left(\xi(t)\right) \left(\mathcal{A}_i e(t) + \mathcal{B}_i f(t)\right) \\ r(t) = \mathcal{C}e(t) + \mathcal{D}f(t) \end{cases}$$

• Frequency of signal f(t): $\omega_f \in \Omega_f^l = \{\omega_f \in \mathbb{R} \mid |\omega_f| \le \omega_f\}$

<u>Lemma 2</u>.

The H_ performance: $||r(t)||_2 > \beta ||f(t)||_2$ is garanteed in the low frequency band $(\omega_f \le \omega_{f_l})$ if there exist symmetric matrices P and Q > 0 such that:

$$\begin{pmatrix} \mathcal{A}_{i} & \mathcal{B}_{i} \\ I & 0 \end{pmatrix}^{T} \begin{pmatrix} -Q & P \\ P & \omega_{f_{i}}^{2}Q \end{pmatrix} \begin{pmatrix} \mathcal{A}_{i} & \mathcal{B}_{i} \\ I & 0 \end{pmatrix}$$
$$+ \begin{pmatrix} \mathcal{C} & \mathcal{D} \\ 0 & I \end{pmatrix}^{T} \begin{pmatrix} -I & 0 \\ 0 & \beta^{2}I \end{pmatrix} \begin{pmatrix} \mathcal{C} & \mathcal{D} \\ 0 & I \end{pmatrix} < 0$$

T-S systems

$$\begin{cases} \dot{x}(t) = A_{\sigma}x(t) + B_{\sigma}u(t) + R_{\sigma}d(t) + F_{\sigma}f(t) \\ y(t) = Cx(t) + Dd(t) + Hf(t) \end{cases}$$

$$\left(M_{_{\sigma}} \ = \ \sum_{i=1}^n \ \mu_i(\xi) M_{_i}
ight)$$

✓ UIO-Multiobjective design : $\begin{cases} \dot{z}(t) = N_{\sigma}z(t) + G_{\sigma}u(t) + L_{\sigma}y(t) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases}$

• Estimation (IUO):
$$\lim_{t \to \infty} (\hat{x}(t) - x(t)) \to 0$$

- Robustnesse to $d(t) \neq 0$: minimise γ such that $||r(t)||_2 < \gamma ||d(t)|_2$ (H ∞ in the middle frequency domain).
- Sensitivity to $f(t) \neq 0$: maximise β such that $||r(t)||_2 > \beta ||f(t)||_2$ (H_ in the low frequency domain).

T-S systems

$$\begin{cases} \dot{x}(t) = A_{\sigma}x(t) + B_{\sigma}u(t) + R_{\sigma}d(t) + F_{\sigma}f(t) \\ y(t) = Cx(t) + Dd(t) + Hf(t) \end{cases}$$

$$\left(M_{_{\sigma}} \,=\, \sum_{i=1}^n \,\mu_i(\xi) M_{_i}
ight)$$

✓ UIO-Multiobjective design : □

$$\begin{cases} \dot{z}(t) = N_{\sigma} z(t) + G_{\sigma} u(t) + L_{\sigma} y(t) \\ \hat{x}(t) = z(t) - E y(t) \end{cases}$$

✓ Aims to achieve:

1- For d=0 and f=0: the filtering error system is asymptotically stable.

2- for $d \neq 0$: the H ∞ performance from d(t) to r(t) is less than a given positive scalar γ in the middle frequency domain.

3- for $f \neq 0$: the H_ performance from f(t) to r(t) is greater than a given positive scalar β in the low frequency domain.

Design Conditions

T-S system:

$$\begin{cases} \dot{x}(t) = A_{\sigma}x(t) + B_{\sigma}u(t) + R_{\sigma}d(t) + F_{\sigma}f(t) \\ y(t) = Cx(t) + Dd(t) + Hf(t) \end{cases}$$

UIO structure:

$$\begin{cases} \dot{z}(t) = N_{\sigma} z(t) + G_{\sigma} u(t) + L_{\sigma} y(t) \\ \hat{x}(t) = z(t) - E y(t) \end{cases}$$

Dynamic of the filtering error:

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i \left(\xi(t)\right) \left(N_i e(t) + (TR_i - K_i D)d(t) + (TF_i - K_i H)f(t)\right)$$

$$r(t) = Ce(t) + Dd(t) + Hf(t)$$

with

$$e(t) = x(t) - \hat{x}(t) \qquad r(t) = y(t) - \hat{y}(t)$$
$$T = I_n + EC \qquad K_i = N_i E + L_i$$

Sensitivity to faults - FFD $\dot{e}(t) = \sum_{i=1}^{r} \mu_i \left(\xi(t)\right) \left(N_i e(t) + (TF_i - K_i H) f(t)\right)$ r(t) = Ce(t) + Hf(t)

• For f(t) \neq 0, H_ performance : $||r(t)||_2 > \beta ||f(t)||_2$

✓ Aims to achieve:

1- For d=0 and f=0: the filtering error system is asymptotically stable.

2- for d≠0: the H∞ performance from d(t) to r(t) is less than a given positive scalar γ in the middle frequency domain.

3- for f≠0: the H_performance from f(t) to r(t) is greater than a given positive scalar β in the low frequency domain.

I Theorem : Sensitivity to faults (H_ performance)

The H_ performance is garanteed in the frequency domaine

$$\begin{pmatrix} -Q & (*) & (*) & (*) \\ P - \Xi & \Theta_f & (*) & (*) \\ 0 & Y_f & -H^T H & (*) \\ 0 & 0 & \beta I & -I \end{pmatrix} < 0$$
 (LMI-S)

with: $\Theta_f = ((\Xi + SC)A_i - W_iC) + ((\Xi + SC)A_i - W_iC)^T - C^TC + \omega_{f_i}^2Q$ $Y_f = ((\Xi + SC)F_i - W_iH)^T - H^TC$

Robustness to disturbances - FFD

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i \left(\xi(t)\right) \left(N_i e(t) + (TR_i - K_i D)d(t)\right)$$

$$r(t) = Ce(t) + Dd(t)$$

• Pour d(t) \neq 0, performance H ∞ : $||r(t)||_2 < \gamma ||d(t)|_2$

1- For d=0 and f=0: the filtering error system is asymptotically stable.

2- for d≠0: the H∞ performance from d(t) to r(t) is less than a given positive scalar γ in the middle frequency domain.

3- for f≠0: the H performance from f(t) to r(t) is greater than a given positive scalar β in the low frequency domain.

Theorem : Robustness to disturbances

The H ∞ performance is garanteed in the frequency domaine $\left[\omega_{d1}, \omega_{d2}\right]$

$$\begin{pmatrix} -Q & (*) & (*) & (*) \\ P - j \omega_d Q - \Xi & \Theta_d & (*) & (*) \\ 0 & Y_d & -\gamma^2 I & (*) \\ 0 & C & D & -I \end{pmatrix} < 0$$
 (LMI-R)

with:
$$\Theta_d = (\Xi + SC)A_i - W_iC + ((\Xi + SC)A_i - W_iC)^T - (\omega_{d_1}\omega_{d_2}Q)^T$$

 $Y_d = ((\Xi + SC)R_i - W_iD)^T$

Observer convergence conditions

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i \left(\xi(t) \right) N_i e(t)$$
$$r(t) = Ce(t)$$

✓ Aims to achieve:

1- For d=0 and f=0: the filtering error system is asymptotically stable.

2- for d≠0: the H∞ performance from d(t) to r(t) is less than a given positive scalar γ in the middle frequency domain.

3- for f≠0: the H performance from f(t) to r(t) is greater than a given positive scalar β in the low frequency domain.

• For d(t)=0 & f(t)=0

Theorem : Asymptotic stability of e(t)

$$\begin{pmatrix} -\Sigma - \Sigma^{T} & (*) & (*) \\ \left((\Sigma + SC)A_{i} - W_{i}C + U \right)^{T} & -U & (*) \\ \Sigma^{T} & 0 & -U \end{pmatrix} < 0 \qquad (LMI-AS)$$

LMI Design conditions : FDD

max
$$\beta$$
, min γ :
 $J_i - (\Sigma + SC)B_i = 0$
 $S[D \quad H] = 0$
(LMI-R), (LMI-S), (LMI-AS)

✓ Aims to achieve:

1- For d=0 and f=0: the filtering error system is asymptotically stable.

2- for d≠0: the H∞ performance from d(t) to r(t) is less than a given positive scalar γ in the middle frequency domain.

3- for f≠0: the H performance from f(t) to r(t) is greater than a given positive scalar β in the low frequency domain.

If these conditions are satisfied, then the observer parameters :

$$E = \sum^{-1} S$$

$$G_{i} = \sum^{-1} J_{i}$$

$$N_{i} = (I + EC)A_{i} - \sum^{-1} W_{i}C$$

$$L_{i} = \sum^{-1} W_{i} - N_{i}E$$

A. Chibani, M. Chadli, P. Shi. Fuzzy Fault Detection Filter Design for T-S Fuzzy Systems in Finite Frequency Domain. **IEEE Transactions on Fuzzy Systems 25 (5), 1051-1061, 2018**.

Illustrative example:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r=2} \mu_i(\zeta(t)) \Big(A_i x(t) + B_i u(t) + R_i d(t) + F_i f(t) \Big) \\ y(t) = C x(t) + D d(t) + H f(t) \end{cases}$$

with

$$A_{1} = \begin{pmatrix} -18.5 & 5 & 18.5 \\ 0 & -20.9 & 15 \\ 18.5 & 15 & -33.5 \end{pmatrix} \qquad A_{2} = \begin{pmatrix} -22.1 & 0 & 22.1 \\ 1 & -23.3 & 17.6 \\ 17.1 & 17.6 & -39.5 \end{pmatrix} \qquad B_{1} = \begin{pmatrix} 1 \\ 0.5 \\ 0.5 \end{pmatrix}$$

$$B_{2} = \begin{pmatrix} 0.5 \\ 1 \\ 0.25 \end{pmatrix} \qquad R_{1} = \begin{pmatrix} 0 \\ 0.6 \\ 0.25 \end{pmatrix} \qquad R_{2} = \begin{pmatrix} 0.25 \\ 0 \\ 0.6 \end{pmatrix} \qquad F_{1} = F_{2} = \begin{pmatrix} 0 \\ 0.6 \\ 0.25 \end{pmatrix}$$
$$C = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 0.3 \\ 0.3 \end{pmatrix} \qquad H = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Illustrative example:

No solution (UIO) with the Full Frequency Domaine approach

Illustrative example:

• Frequency of signal
$$d(t)$$
: $\omega_d \in \Omega_d^m = \{\omega_d \in \mathbb{R} \mid \omega_{d_1} = 0.5 \le \omega_d \le \omega_{d_2} = 1\}$
• Frequency of signal $f(t)$: $\omega_f \in \Omega_f^l = \{\omega_f \in \mathbb{R} \mid |\omega_f| \le \omega_{f_1} = 0.3\}$

Solution : $\gamma = 0.4401$, $\beta = 1.4106$ $N_{1} = \begin{pmatrix} -48.5957 & -20.4247 & -15.2247 \\ 13.5244 & -8.6546 & 7.8692 \\ -0.8131 & -0.6916 & -2.2464 \end{pmatrix} \qquad L_{1} = \begin{pmatrix} -10.1464 & 8.8850 \\ -16.4256 & 18.0037 \\ -0.6673 & 0.6274 \end{pmatrix}$ $N_{2} = \begin{pmatrix} -49.7451 & -29.8316 & -12.3870 \\ 13.4709 & -8.9321 & 9.8851 \\ -0.7446 & -0.8115 & -2.0202 \end{pmatrix} \qquad L_{2} = \begin{pmatrix} -29.8558 & 28.5168 \\ -14.4456 & 15.9344 \\ -1.1722 & 1.1284 \end{pmatrix}$ $G_{1} = \begin{pmatrix} 0.1525 \\ 0.9082 \\ -0.0231 \end{pmatrix} \qquad G_{2} = \begin{pmatrix} 0.0763 \\ 1.2041 \\ -0.0115 \end{pmatrix} \qquad E = \begin{pmatrix} 1.6950 & -1.6950 \\ -0.8164 & 0.8164 \\ 1.0461 & -1.0461 \end{pmatrix}$

Illustrative example:

Sensitivity to faults: without H_

Sensitivity to faults: with H_

A. Chibani, M. Chadli, S.X. Ding, NB Braiek. Design of robust fuzzy fault detection filter for polynomial fuzzy systems with new finite frequency specifications. **Automatica 93, 42–54, 2018**.

Perspectives

Multi-Agents Systems

Event-Triggered Control/Estimation for Cyber-Physical Systems

Distributed FTC/FDI

□ Fractional-order systems

Agent - i : LPV, T-S

- Related Personal References (FFD) :
 - ✓ A. Chibani, <u>M. Chadli</u>, S.X. Ding, NB Braiek. Design of robust fuzzy fault detection filter for polynomial fuzzy systems with new finite frequency specifications. **Automatica 93, 42–54, 2018**.
 - ✓ A. Chibani, <u>M. Chadli</u>, P. Shi. Fuzzy Fault Detection Filter Design for T-S Fuzzy Systems in Finite Frequency Domain. IEEE Transactions on Fuzzy Systems 25 (5), 1051-1061, 2018.
 - ✓ S Marir, <u>M Chadli</u>, MV Basin. Bounded real lemma for singular linear continuous-time fractional-order systems. Automatica 35, 2022.

