The Institution of Engineering and Technology

IET Coventry and Warwickshire Local Network,
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Fuzzy Modeling, Estimation Techniques and their Applications

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Outline

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PARIS-SACLAY

Contexte and Motivation

- Artificial Intelligence: Fuzzy and Soft Computing
- Fuzzy/Takagi-Sugeno modeling

Estimation

- Unknown Input Fuzzy Observers
- Limitations: Full frequency approach

Estimation: Finite Frequency Domain Approach

- Objectives & Motivation
- Multiobjective synthesis: Sensitivity & Robustness

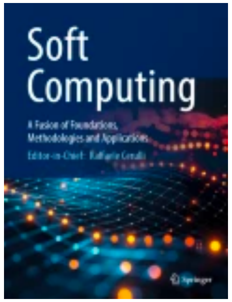
Perspectives

Contexte & Motivation

Artificial Intelligence: Fuzzy Logic and Soft Computing

- ❑ Soft computing includes fuzzy logic, neural networks and meta-heuristic algorithms, as well as their hybrid combinations.

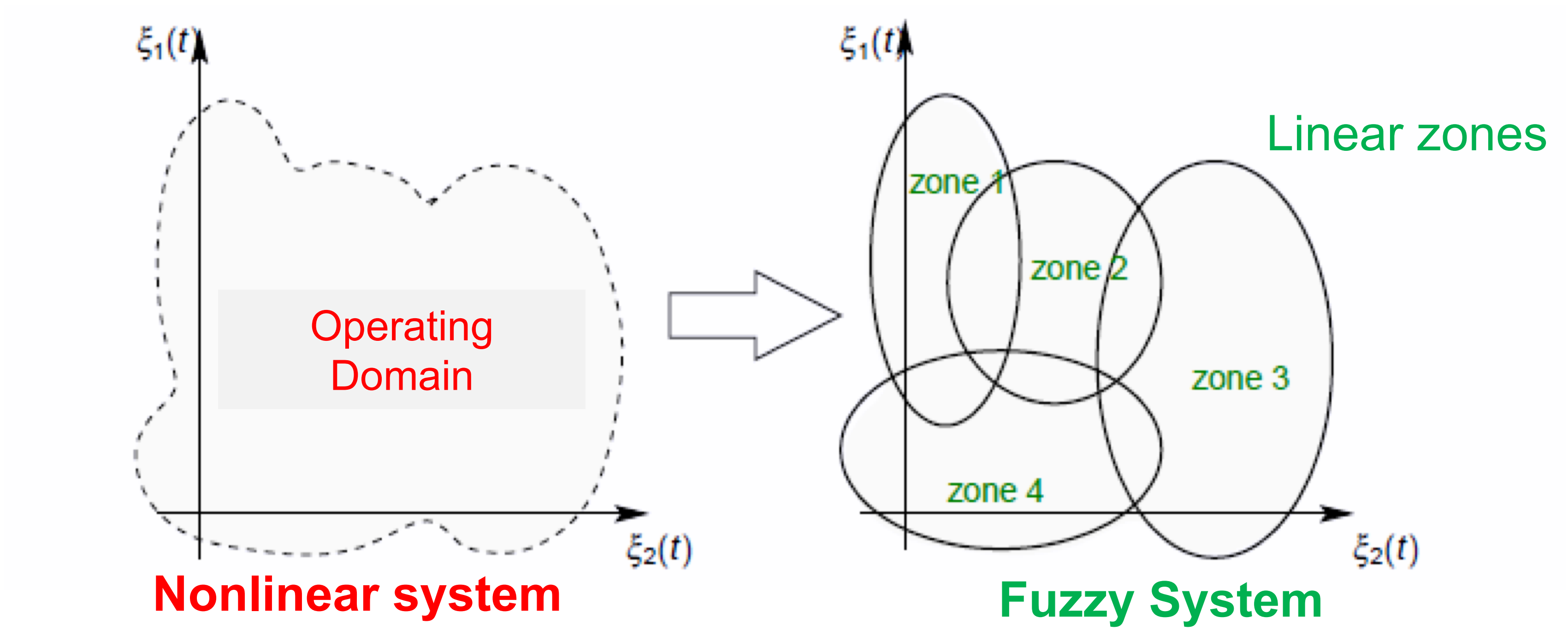
- ❑ Modern artificial intelligence includes the theoretical developments and applications of soft computing techniques.



- Dubois, D., Prade, H. **Soft computing, fuzzy logic, and artificial intelligence**. *Soft Computing* Volume 2, 1998.

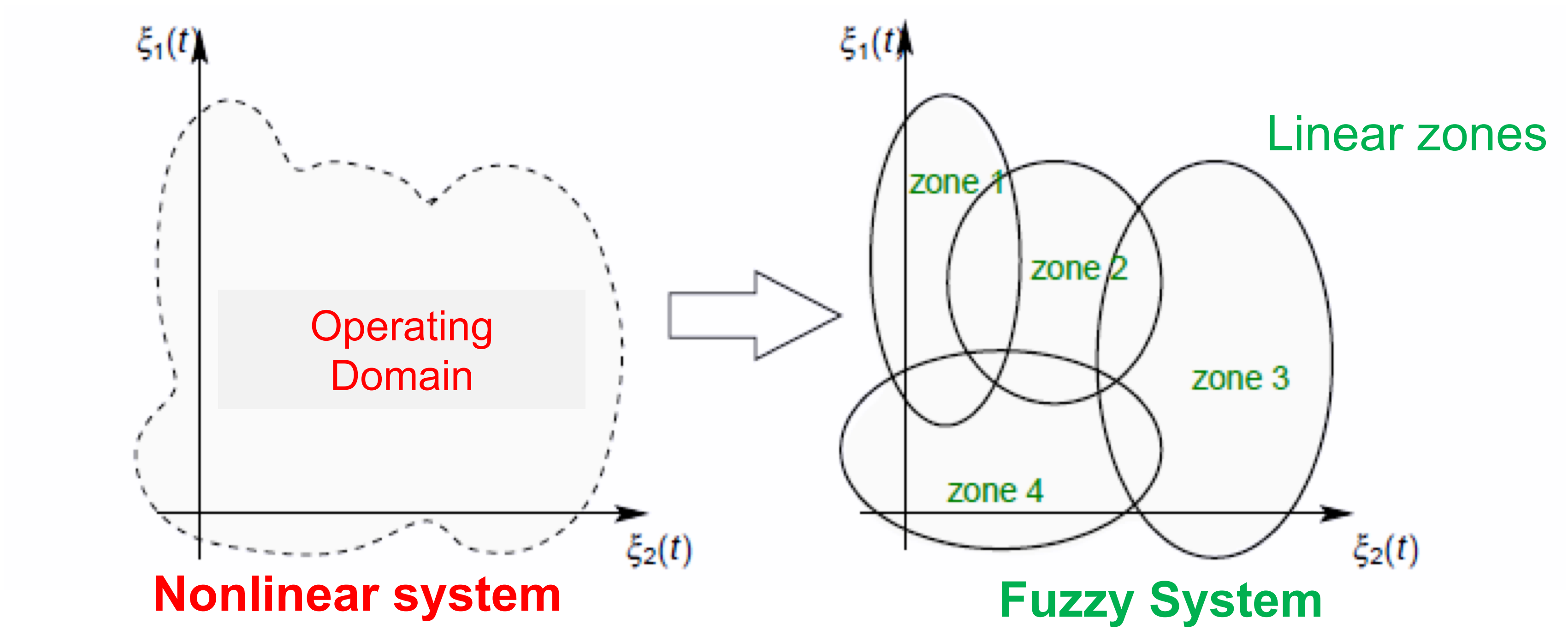
- Ronald R. Yager **Fuzzy Sets and Systems Fuzzy logics and artificial intelligence**. Fuzzy Sets and Systems Volume 90, Issue 2, 1997.

Fuzzy/Nonlinear Systems



- L. Zadeh. Outline of a new approach to the analysis of complex system and decision process. IEEE transaction on Systems Man and Cybernetic-part C, 3(1), p. 28-44, 1973.

Fuzzy/Nonlinear Systems



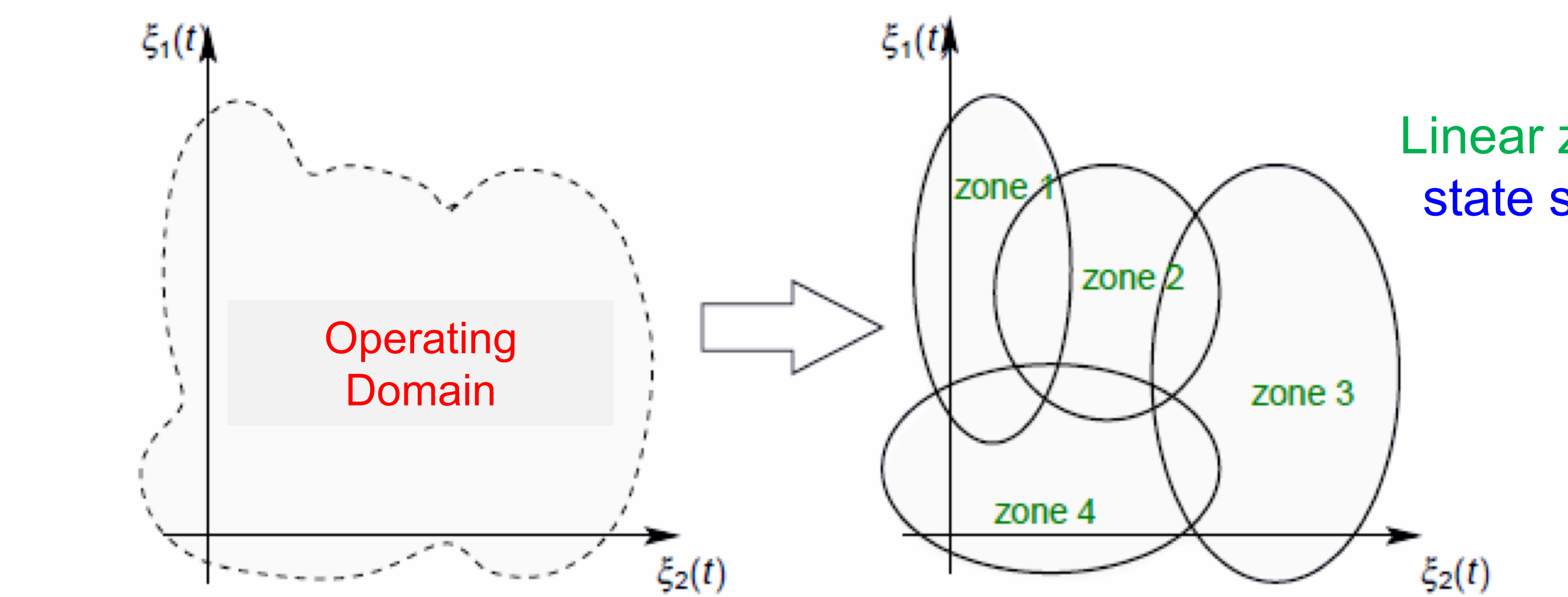
Objectives

- **Numerical tools : LMI & SOS**
- **Reduction of conservatism**
- **Certification: Control/Estimation of design algorithms**

Fuzzy/Nonlinear Systems : T-S Fuzzy Systems

$$\begin{cases} \sigma x(t) = f(x(t), u(t)) \\ y(t) = h(x(t)) \end{cases} \Rightarrow \begin{cases} \sigma x(t) = \sum_{i=1}^{n} \mu_i(\xi)\left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum_{i=1}^{n} \mu_i(\xi) C_i x(t) \end{cases}$$

$\sigma x(t) = \dot{x}(t)$ or $x(t+1)$



Nonlinear system

Linear zones: state space

T-S System

[Multiple models (LTI)]

Fuzzy/Nonlinear Systems : T-S Fuzzy Systems

$$\begin{cases} \sigma x(t) = f(x(t), u(t)) \\ y(t) = h(x(t)) \end{cases} \Rightarrow \begin{cases} \sigma x(t) = \sum_{i=1}^{n} \mu_i(\xi)\Big(A_i(x)x(t) + B_i(x)u(t)\Big) \\ y(t) = \sum_{i=1}^{n} \mu_i(\xi) C_i(x)x(t) \end{cases}$$

Nonlinear Systems ---- > Polynomial T-S Systems

Polynomial T-S Systems : SOS Tool

- T. Takagi, M. Sugeno. Fuzzy identification of systems and its application to modelling and control. IEEE Trans. on Systems, Man, Cybernetics, vol. 15, no.1, p. 116-132,1985.

Fuzzy/Nonlinear Systems : T-S Fuzzy, LPV, ..

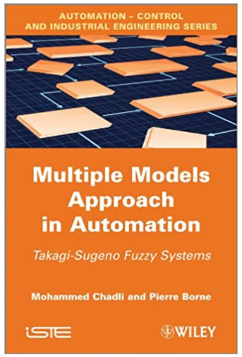
$$\begin{cases} \sigma x(t) = f(x(t), u(t)) \\ y(t) = h(x(t)) \end{cases} \Rightarrow \begin{cases} \sigma x(t) = \sum_{i=1}^{n} \xi_i(\theta)\left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum_{i=1}^{n} \xi_i(\theta) C_i x(t) \end{cases}$$

$$\sum_{i=1}^{n} \xi_i(\theta) = 1, \quad \xi_i(\theta) \geq 0$$

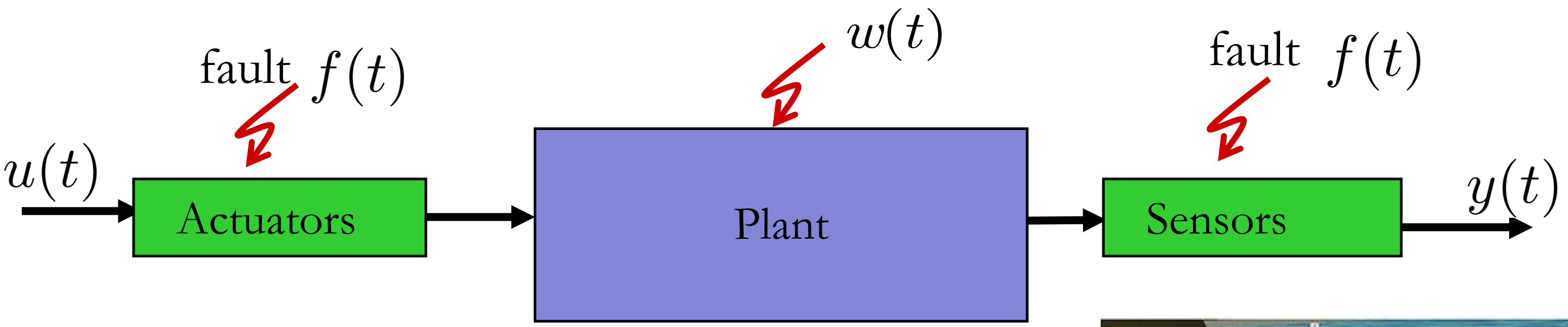
LPV	T-S	Switching
$\theta \to \rho(t)$	$\theta \to x(t), \rho(t)\ldots$	$\xi_i(\theta) \in \{0,1\}$

1. **Controller & Observer Design**
2. **FDI-FTC**
   - **Numerical tools : LMI & SOS**
   - **Reduction of conservatism of the design problems**

M. Chadli & P. Borne. **Multiple Models Approach in Automation: Takagi-Sugeno Fuzzy Systems.** Wiley. p. 208. 2013.



Fuzzy/Nonlinear Systems : Contexte



- ***f(t)*** : fault/sensor actuators
- ***w(t)*** : disturbances

Objectives:

- **Estimation/Control of nonlinear systems**
- **Fault Detection and Isolation of faults** (estimation?)
- **How to maintain/enable a system to continue operating properly in presence of faults (FTC) ?**
  - ↳ Avoid dangerous situations

LMI design conditions

TS Systems :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{n} \mu_i(\xi)\Big(A_i x(t) + B_i u(t) + R_i \overline{u}(t)\Big) \\ y(t) = Cx(t) + F\overline{u}(t) \end{cases}$$

Unknown Input Observer (UIO) :

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{n} \mu_i(\xi)\Big(N_i z(t) + G_i u(t) + L_i y(t)\Big) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases}$$

Objective : $N_i,\ G_i,\ L_i,\ E\ ?$

$$s.t. \quad \lim_{t\to\infty}\Big(\hat{x}(t) - x(t)\Big) \to 0$$

LMI design conditions

$$\begin{cases} XA_i + SCA_i - W_iC + (XA_i + SCA_i - W_iC) < 0 \\ SF = 0 \\ \left(X + SC\right)R_i = W_iF \end{cases}$$

Observer parameters:

$$\begin{cases} E = X^{-1}S, \\ G_{i1} = \left(I + EC\right)B_i \\ N_i = \left(I + EC\right)A_i - X^{-1}W_iC \\ L_i = X^{-1}W_i - N_iE \end{cases}$$

Performances : LMI regions

LMI design conditions

Extensions : Unmeasurables variables, Disturbances

1. Unmesurable variables : continuous/discrete-time cases

$\mu_i(\theta) \Rightarrow \mu_i(\hat{\theta})$:

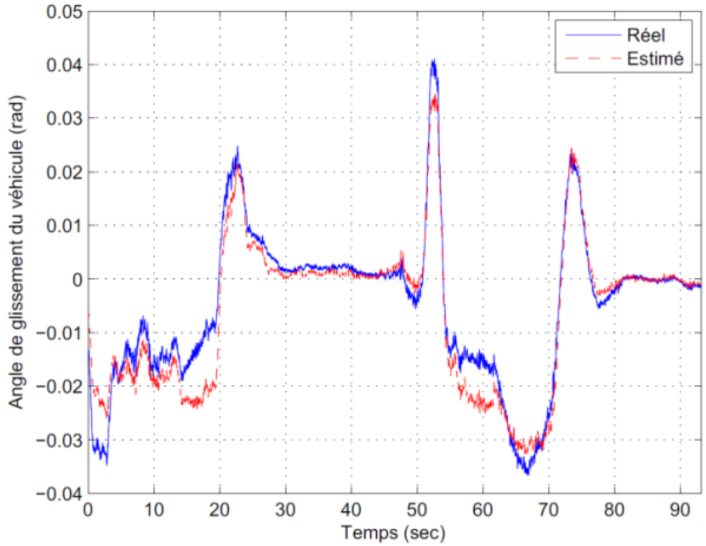
$$\begin{cases} \sigma x(t) = \sum_{i=1}^{n} \mu_i(\hat{\xi})\Big(A_i x(t) + B_i u(t) + R_i \overline{u}(t) + H_i w(t)\Big) \\ y(t) = \sum_{i=1}^{n} \mu_i(\hat{\xi})\Big(C_i x(t) + F_i \overline{u}(t) + J_i w(t)\Big) \end{cases}$$

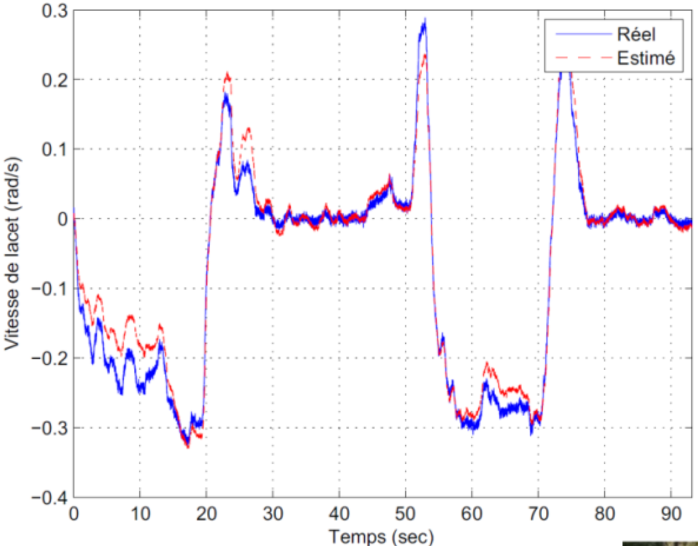
2. Polynomial TS system (SOS) :

$$\begin{cases} \sigma x(t) = \sum_{i=1}^{n} \mu_i(\xi)\Big(A_i(x) x(t) + B_i(x) u(t) + R_i(x) \overline{u}(t)\Big) \\ y(t) = Cx(t) + F\overline{u}(t) + Jw(t) \end{cases}$$

Vehicle application: Experimental validation

Estimation of variables (lateral dynamic)

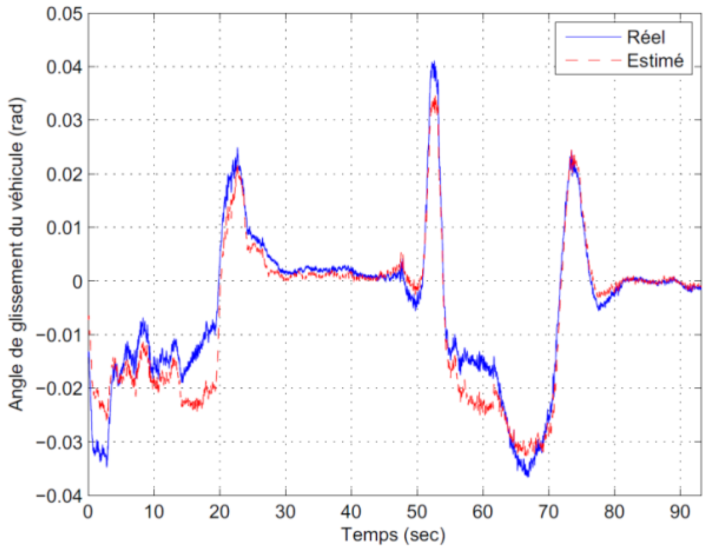


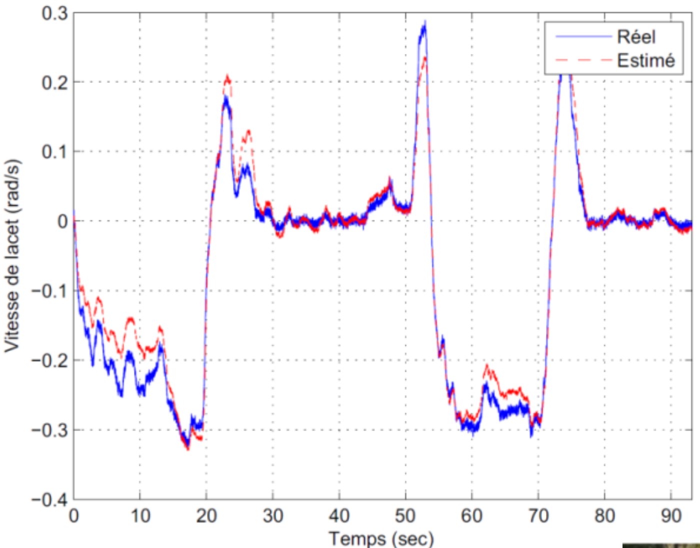


*Vehicle dynamics and road curvature estimation for lane departure warning system using robust fuzzy observers: **Experimental validation**.*
Vehicle System Dynamics Journal, 2015. *(PhD thesis of H. Dahmani)*

Vehicle application: Experimental validation

Estimation of variables
(lateral dynamic)





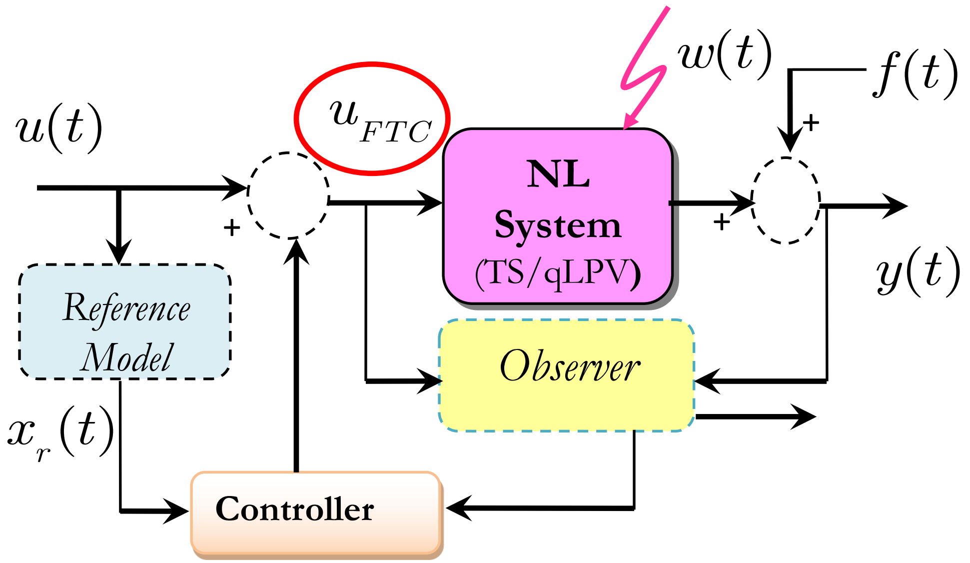
*Vehicle dynamics and road curvature estimation for lane departure warning system using robust fuzzy observers: **Experimental validation**.*
***Vehicle System Dynamics Journal, 2015**. (PhD thesis of H. Dahmani)*

Cryptography : Chaos synchronisation

*M. Chadli, I. Zelinka, T. Youssef. Unknown inputs observer design for fuzzy systems with application to chaotic system reconstruction. **Computers & Mathematics with Applications** 66 (2), 147-154. 2016*

How Maintaining Security in the Presence of Faults: Ref. model

- **Integration of contraintes s.t. delays**
- **Reconfiguration of control laws**

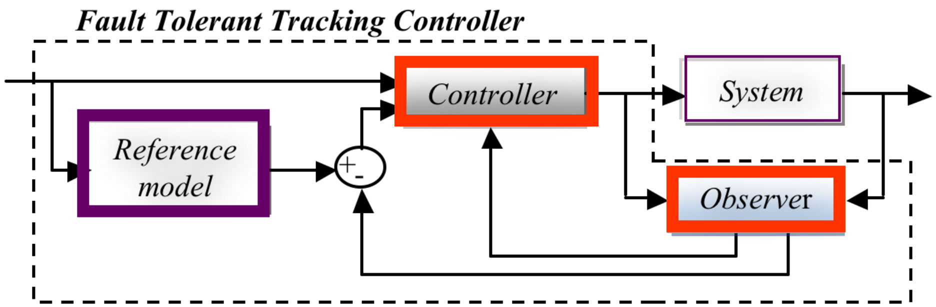


$$u_{FTC}(t) = u(t) + \sum_{j=1}^{r} h_j\left(\hat{\xi}(t)\right)\left(K_j\left(\hat{x}(t) - x_r(t)\right)\right)$$

D Saifia, M Chadli, et al. Robust H∞ static output feedback control for discrete fuzzy systems with actuator saturation via fuzzy Lyapunov functions. Asian Journal of Control, 2019.

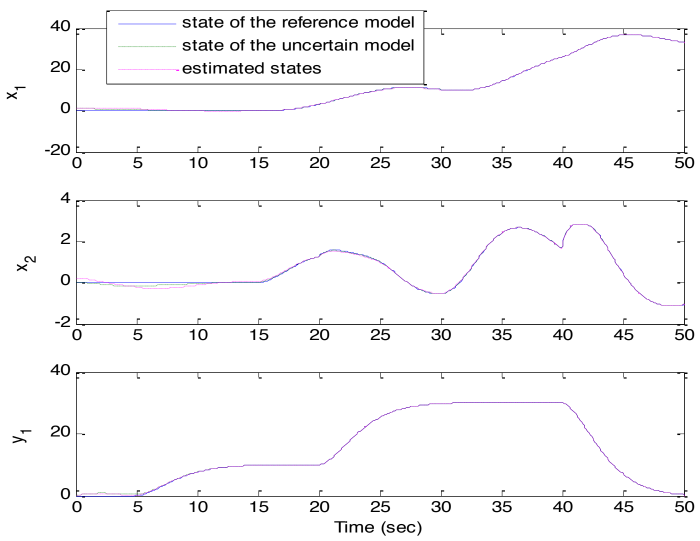
Vehicle application

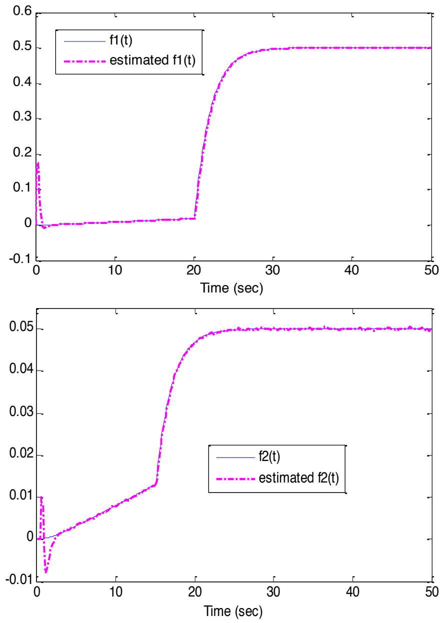
Example of FTCS strategy: VTOL (Vertical Takeoff and Landing)



Tracking fault tolerant controller design methodology

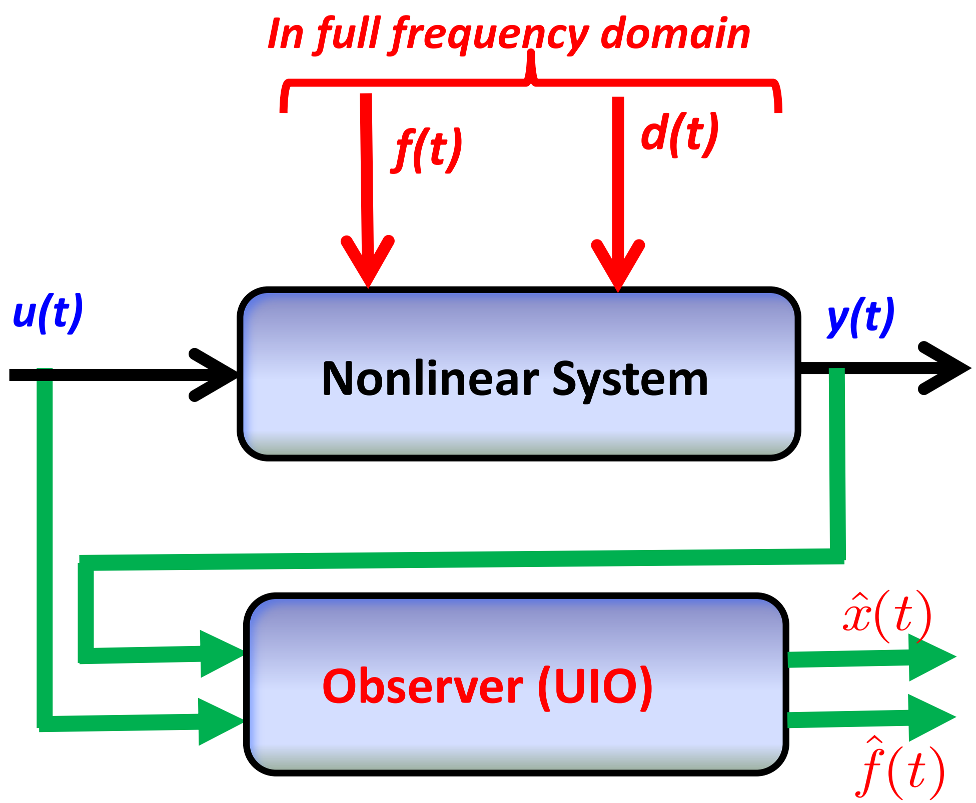
➢ Actuator faults (VTOL)





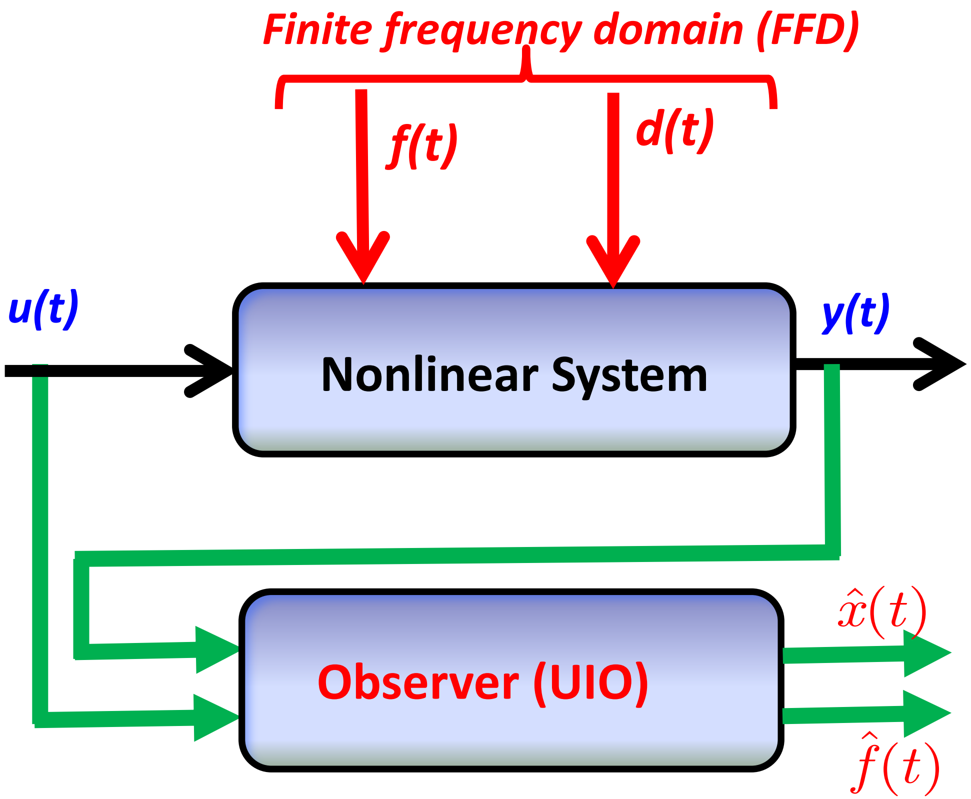
M. Chadli, S. Aouaouda , P. Shi. Robust fault tolerant tracking controller design for a VTOL aircraft. **Journal of the Franklin Institute** Volume 350, Issue 9, 2013.

- **Findings :**
  - ✓ **Observer for full frequency domaine only**
  - ✓ **Results do not take into account the frequency domain of the signals: Conservatism**



Objectives

- ❑ **Incorporate the frequency range of the external signals into the synthesis conditions.**
- ❑ **Robust Estimation - Finite Frequency Domain (FFD)**

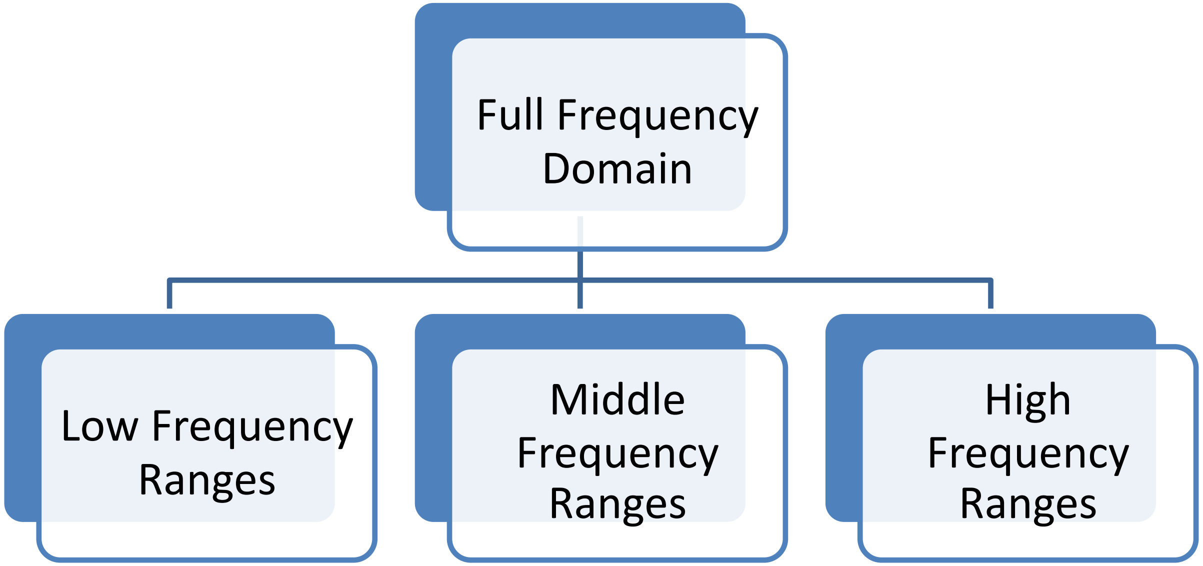


The finite frequency approach consists on incorporating the frequency range of the external signals in the design conditions

PhD thesis of A. Chibani : **IEEE-TFS 2018, Automatica 2018**

Finite Frequency Domain Approach

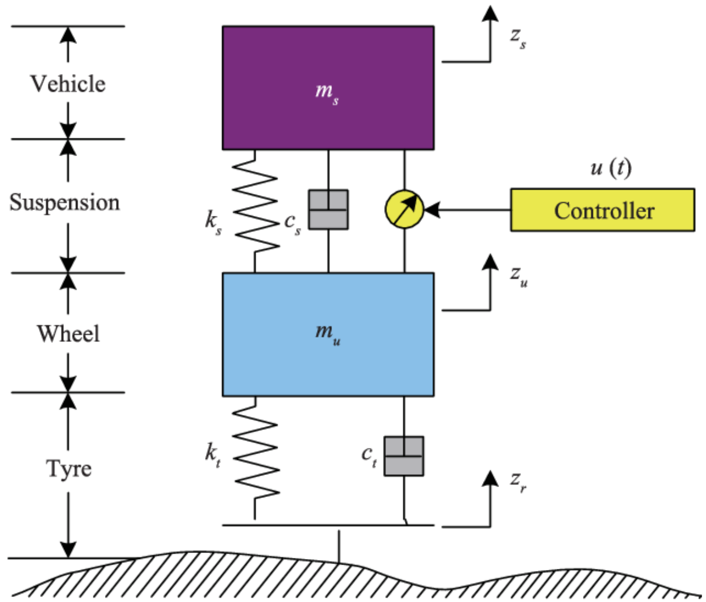
- The finite frequency method generally leads to more efficient results and less conservative conditions.



according to the frequency of the external signal

T. Iwasaki, S. Hara. Generalized KYP lemma: unified frequency domain inequalities. IEEE TAC 2005.

Motivation : Vertical vibrations in frequencies between 4 Hz and 8 Hz are the most sensitive range for the human body. (Norm ISO2361)



Objectives :

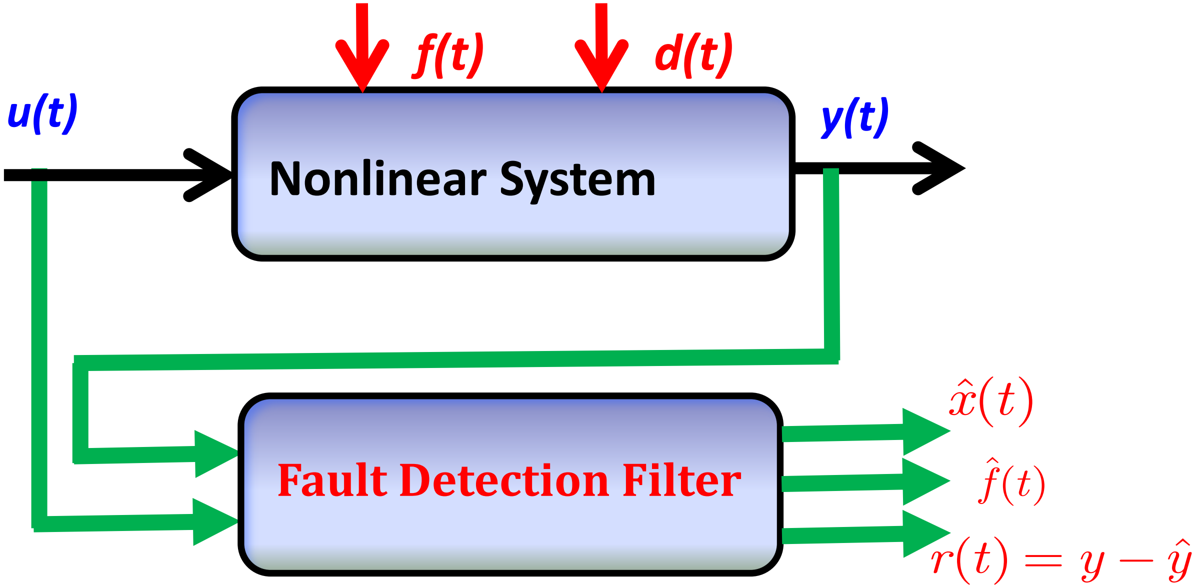
- ❑ **Incorporate the frequency range of the external signals into the synthesis conditions.**
- ❑ **Robust Estimation - Finite Frequency Domain (FFD)**

➢ **Multiobjective FDI:**

H∞ : $\|r(t)\|_2 < \gamma \|d(t)\|_2$

H_ : $\|r(t)\|_2 > \beta \|f(t)\|_2$

Estimation: $\hat{x}(t),\ \hat{f}(t)$



- **T-S systems**

$$\begin{cases} \dot{x}(t) = A_\sigma x(t) + B_\sigma u(t) + R_\sigma d(t) + F_\sigma f(t) \\ y(t) = Cx(t) + Dd(t) + Hf(t) \end{cases}$$

✓ **UIO-Multiobjective design :**

$$\begin{cases} \dot{z}(t) = N_\sigma z(t) + G_\sigma u(t) + L_\sigma y(t) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases}$$

- Estimation (IUO) : $\lim_{t\to\infty}\left(\hat{x}(t) - x(t)\right) \to 0$
- Robustnesse to d(t)≠0 : minimise γ such that $\|r(t)\|_2 < \gamma\|d(t\|_2$ (H∞ in the middle frequency domain).
- Sensitivity to f(t)≠0 : maximise β such that $\|r(t)\|_2 > \beta\|f(t)\|_2$ (H_ in the low frequency domain).

Multiobjective synthesis : FFD-UIO

- **T-S systems**

$$\begin{cases} \dot{x}(t) = A_\sigma x(t) + B_\sigma u(t) + R_\sigma d(t) + F_\sigma f(t) \\ y(t) = Cx(t) + Dd(t) + Hf(t) \end{cases}$$

✓ **UIO-Multiobjective design :**

$$\begin{cases} \dot{z}(t) = N_\sigma z(t) + G_\sigma u(t) + L_\sigma y(t) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases}$$

✓ **Aims to achieve:**

1- For d=0 and f=0: the filtering error system is **asymptotically stable**.

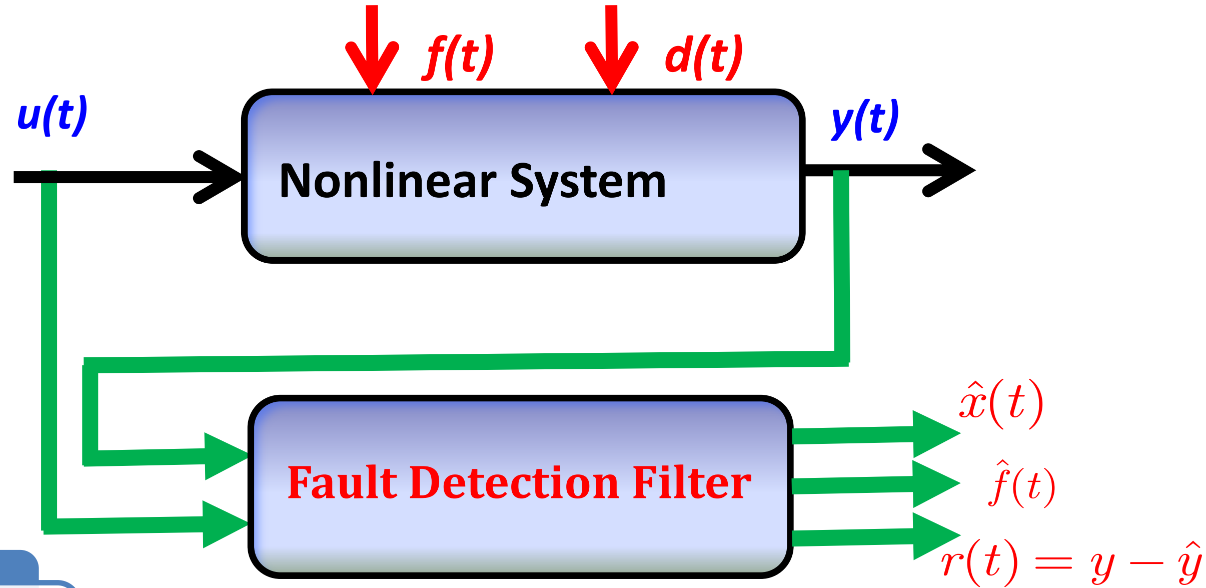
2- for d≠0: the H∞ performance from *d(t)* to *r(t)* is less than a given positive scalar γ (in the middle frequency domain).

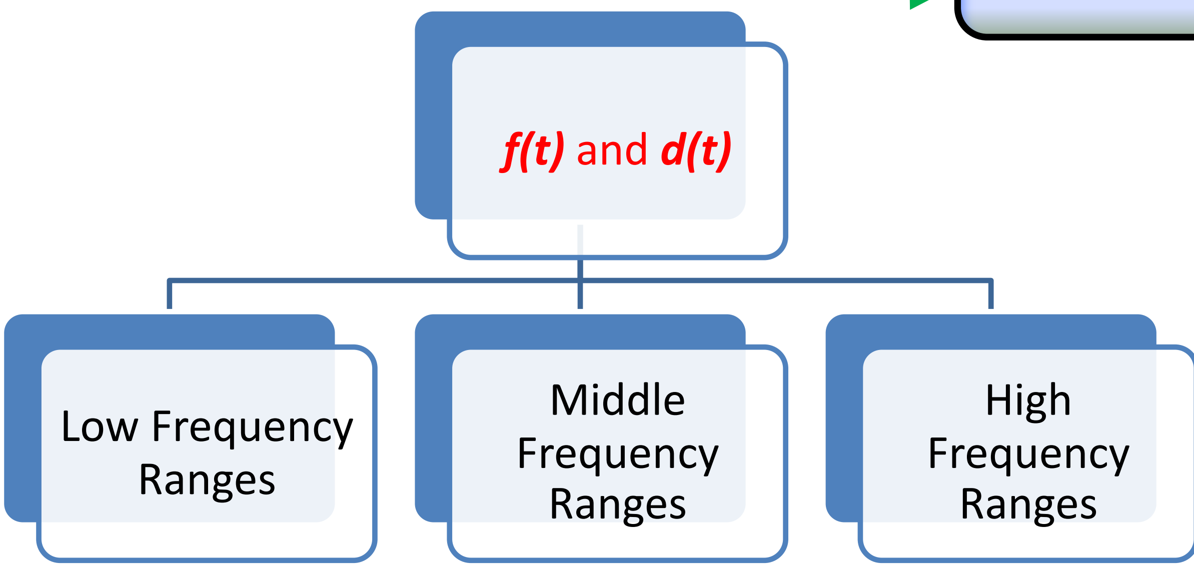
3- for f≠0: the H_ performance from *f(t)* to *r(t)* is greater than a given positive scalar β (in the low frequency domain).

Multiobjective synthesis : FFD

Multiobjectives design:

- ❑ Incorporate the frequency range
- ❑ Robust Estimation in FFD



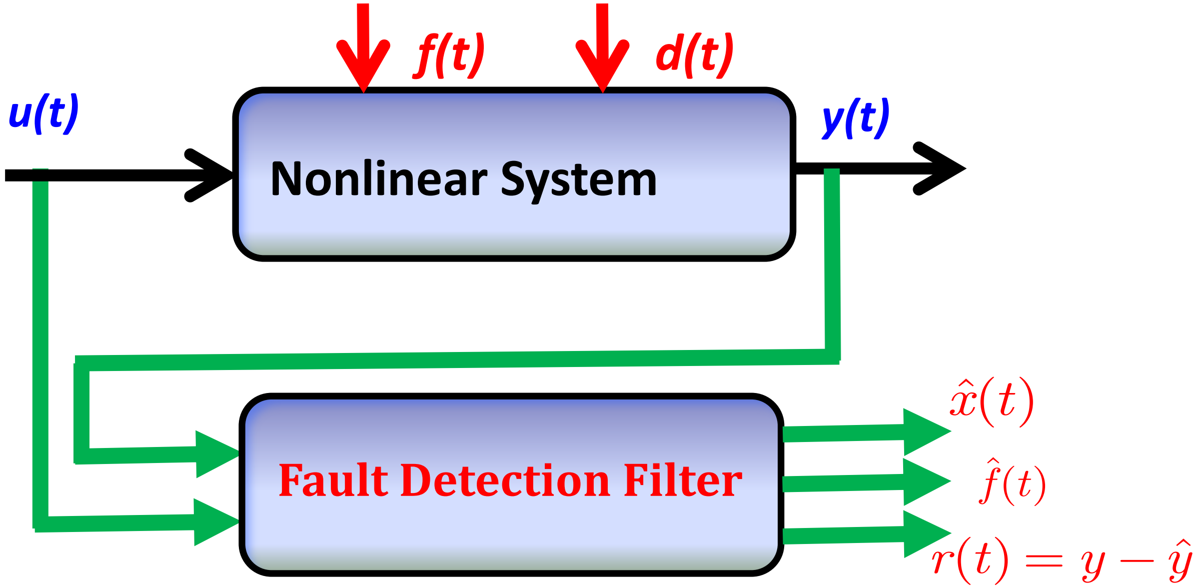


A. Chibani, M. Chadli, P. Shi. Fuzzy Fault Detection Filter Design for T-S Fuzzy Systems in Finite Frequency Domain. **IEEE Transactions on Fuzzy Systems 25 (5), 1051-1061, 2018**.

Illustrative example:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r=2} \mu_i(\zeta(t))\left(A_i x(t) + B_i u(t) + R_i d(t) + F_i f(t)\right) \\ y(t) = Cx(t) + Dd(t) + Hf(t) \end{cases}$$

- Frequency of signal *d(t)* : $\omega_d \in \Omega_d^m = \{\omega_d \in \mathbb{R} \mid \omega_{d_1} = 0.5 \leq \omega_d \leq \omega_{d_2} = 1\}$
- Frequency of signal *f(t)* : $\omega_f \in \Omega_f^l = \{\omega_f \in \mathbb{R} \mid |\omega_f| \leq \omega_{f_l} = 0.3\}$



Multiobjective synthesis : FFD-UIO

Illustrative example:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r=2} \mu_i(\zeta(t))\Big(A_i x(t) + B_i u(t) + R_i d(t) + F_i f(t)\Big) \\ y(t) = Cx(t) + Dd(t) + Hf(t) \end{cases}$$

with

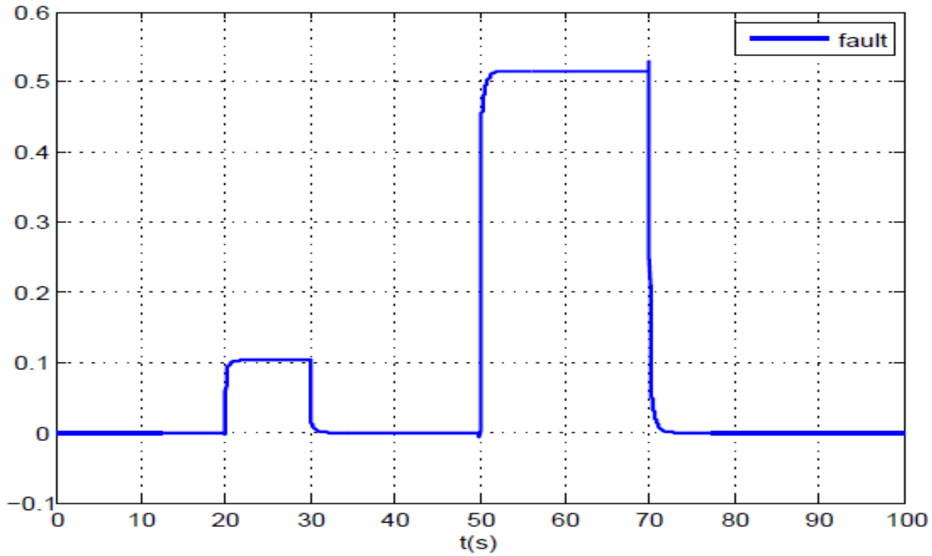
$$A_1 = \begin{pmatrix} -18.5 & 5 & 18.5 \\ 0 & -20.9 & 15 \\ 18.5 & 15 & -33.5 \end{pmatrix} \quad A_2 = \begin{pmatrix} -22.1 & 0 & 22.1 \\ 1 & -23.3 & 17.6 \\ 17.1 & 17.6 & -39.5 \end{pmatrix} \quad B_1 = \begin{pmatrix} 1 \\ 0.5 \\ 0.5 \end{pmatrix}$$

$$B_2 = \begin{pmatrix} 0.5 \\ 1 \\ 0.25 \end{pmatrix} \quad R_1 = \begin{pmatrix} 0 \\ 0.6 \\ 0.25 \end{pmatrix} \quad R_2 = \begin{pmatrix} 0.25 \\ 0 \\ 0.6 \end{pmatrix} \quad F_1 = F_2 = \begin{pmatrix} 0 \\ 0.6 \\ 0.25 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 0.3 \\ 0.3 \end{pmatrix} \quad H = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Illustrative example:

Fault f(t)



No solution (UIO) with the Full Frequency Domaine approach

Illustrative example:

- Frequency of signal *d(t)* : $\omega_d \in \Omega_d^m = \{\omega_d \in \mathbb{R} \mid \omega_{d_1} = 0.5 \leq \omega_d \leq \omega_{d_2} = 1\}$
- Frequency of signal *f(t)* : $\omega_f \in \Omega_f^l = \{\omega_f \in \mathbb{R} \mid |\omega_f| \leq \omega_{f_l} = 0.3\}$

Solution : $\gamma = 0.4401, \quad \beta = 1.4106$

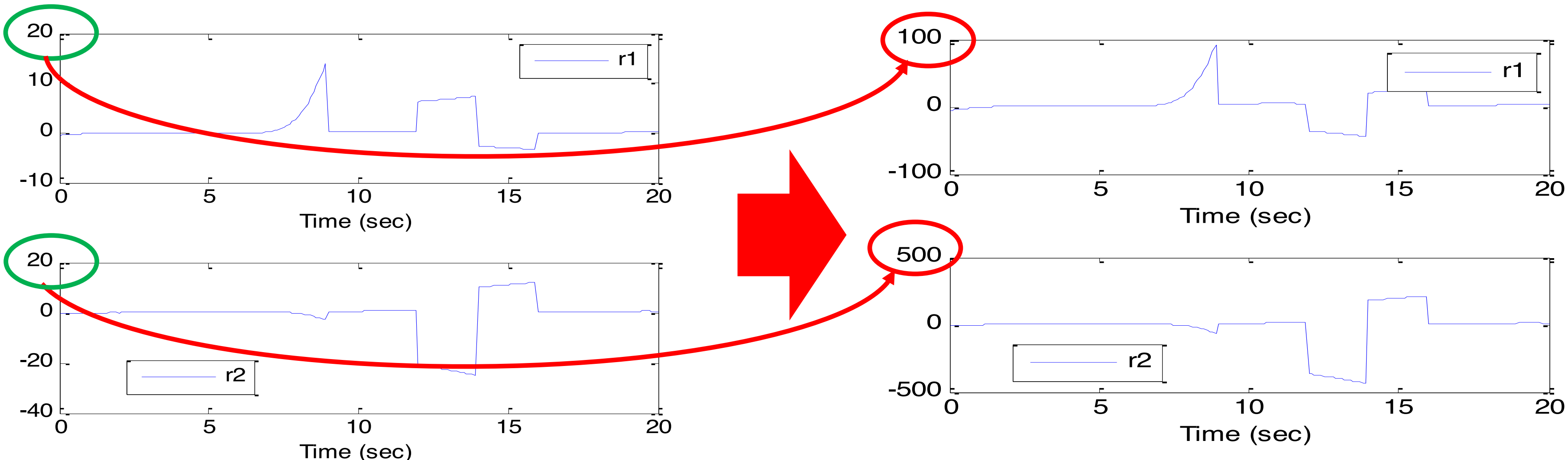
$$N_1 = \begin{pmatrix} -48.5957 & -20.4247 & -15.2247 \\ 13.5244 & -8.6546 & 7.8692 \\ -0.8131 & -0.6916 & -2.2464 \end{pmatrix} \qquad L_1 = \begin{pmatrix} -10.1464 & 8.8850 \\ -16.4256 & 18.0037 \\ -0.6673 & 0.6274 \end{pmatrix}$$

$$N_2 = \begin{pmatrix} -49.7451 & -29.8316 & -12.3870 \\ 13.4709 & -8.9321 & 9.8851 \\ -0.7446 & -0.8115 & -2.0202 \end{pmatrix} \qquad L_2 = \begin{pmatrix} -29.8558 & 28.5168 \\ -14.4456 & 15.9344 \\ -1.1722 & 1.1284 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} 0.1525 \\ 0.9082 \\ -0.0231 \end{pmatrix} \qquad G_2 = \begin{pmatrix} 0.0763 \\ 1.2041 \\ -0.0115 \end{pmatrix} \qquad E = \begin{pmatrix} 1.6950 & -1.6950 \\ -0.8164 & 0.8164 \\ 1.0461 & -1.0461 \end{pmatrix}$$

Illustrative example:

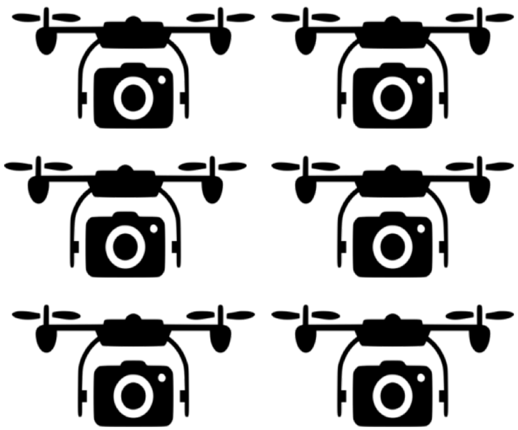
Sensitivity to faults: without H_ | **Sensitivity to faults: with H_**

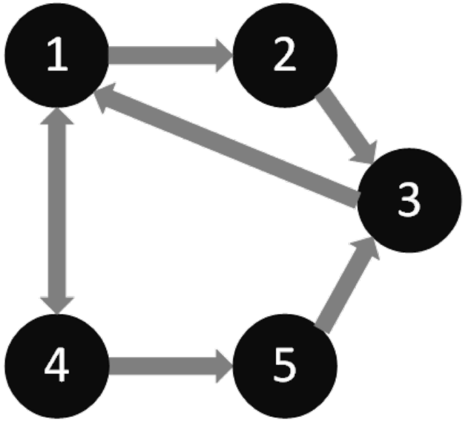


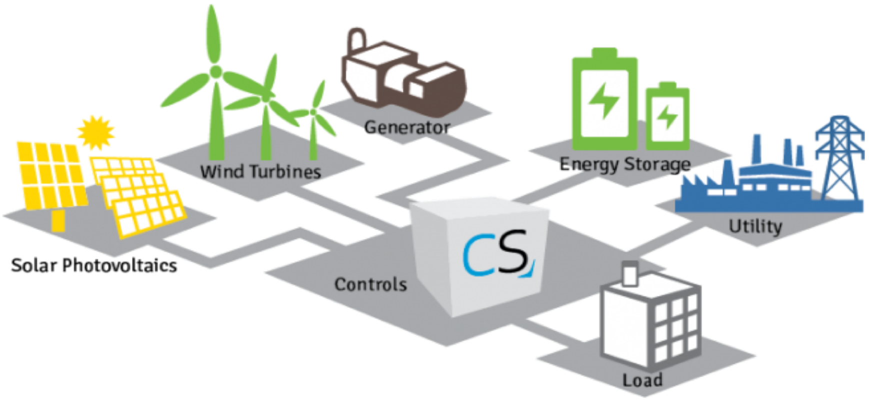
maximisation of sensitivity (with H_)

A. Chibani, M. Chadli, S.X. Ding, NB Braiek. Design of robust fuzzy fault detection filter for polynomial fuzzy systems with new finite frequency specifications. **Automatica 93, 42–54, 2018**.

- ❑ Multi-Agents Systems : UAV/Drones
- ❑ Event-Triggered Esimation/Control for Cyber-Physical Systems
- ❑ Distributed FTC/FDI
- ❑ Fractional-order systems





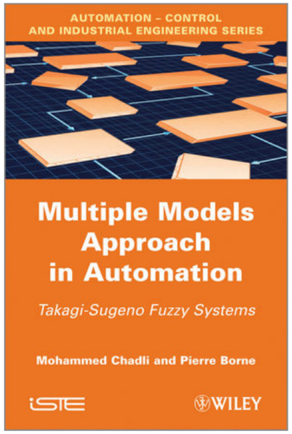


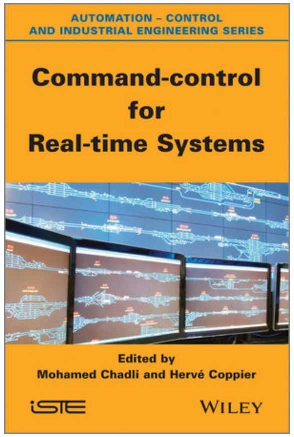
- Related Personal References :

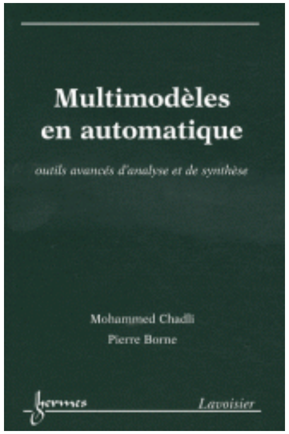
✓A. Chibani, M. Chadli, S.X. Ding, NB Braiek. Design of robust fuzzy fault detection filter for polynomial fuzzy systems with new finite frequency specifications. **Automatica 93, 42–54, 2018**.

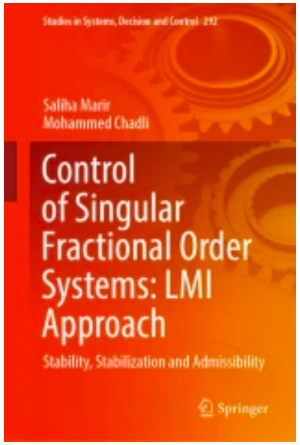
✓A. Chibani, M. Chadli, P. Shi. Fuzzy Fault Detection Filter Design for T-S Fuzzy Systems in Finite Frequency Domain. **IEEE Transactions on Fuzzy Systems 25 (5), 1051-1061, 2018.**

✓S Marir, M Chadli, MV Basin. Bounded real lemma for singular linear continuous-time fractional-order systems. **Automatica 35, 2022.**









Thank you for your attention

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Fuzzy Modeling, Estimation Techniques and their Applications

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